

The hybrid curvaton

Konstantinos Dimopoulos

Consortium for Fundamental Physics, Physics Department, Lancaster University, Lancaster LA1 4YB, UK

Kazunori Kohri

*Cosmophysics group, Theory Center, IPNS, KEK,
and The Graduate University for Advanced Study (Sokendai), Tsukuba 305-0801, Japan*

Tomohiro Matsuda

*Physics Department, Lancaster University, Lancaster LA1 4YB, UK,
and Laboratory of Physics, Saitama Institute of Technology, Fukaya, Saitama 369-0293, Japan*

We study both oscillating and inflating curvaton scenarios when the curvaton mechanism is caused by a hybrid potential. The source of the curvature perturbation is the inhomogeneous phase transition that causes the modulation of the onset of the oscillation. For the supergravity-motivated curvaton there is a possibility of finding natural coincidence of the energy density that is needed to affect non-Gaussianity in the curvaton scenario.

PACS numbers: 98.80Cq

I. INTRODUCTION

The primordial perturbation in the inflationary epoch is widely believed to be the primary origin of the formation of the large-scale structure in the Universe [1]. The generation of the curvature perturbation ζ in the original inflation scenario is due to the inflaton perturbation $\delta\phi$ that exits the horizon during inflation. The curvature perturbation ζ in the original single-field inflation scenario is generated from the inflaton perturbation $\delta\phi$, which remains constant after the horizon exit. In the curvaton scenario, ζ is instead generated after inflation from the isocurvature perturbation of a curvaton field [2] (for a recent review see Ref. [3]).

In this paper we consider a hybrid potential for the curvaton scenario and study both the original (oscillating) [2] and the inflating curvaton [4].¹ The hybrid potential is very important in cosmological model building and a vast variety of models have been considered [7, 8]. This paper presents the first attempt in choosing the hybrid potential $V(\varphi, \sigma)$ for the curvaton mechanism. In this paper we try to keep the discussion as simple as possible. We introduce three parameters α , β and γ to measure the model dependence of the scenario, and avoid highly model-dependent arguments. Since the non-trivial evolution after the waterfall [6] may ruin the simplicity of the model, we assume matter-like evolution ($\rho_\sigma \propto a^{-3}$) that starts just after the phase transition. The phase transition is called “trigger” in this paper because the phase transition triggers the oscillation of the waterfall field σ . A crucial difference from the usual curvaton is that the source of the perturbation is not the perturbation of the oscillating (waterfall) field (because $\delta\sigma = 0$

for large scale) but the perturbation of the slow-rolling field ($\delta\varphi \neq 0$) [9], which is not the inflaton (ϕ) of the primordial inflation but it is the field which triggers the phase transition. The characteristics of this scenario are shown in Fig. 1.

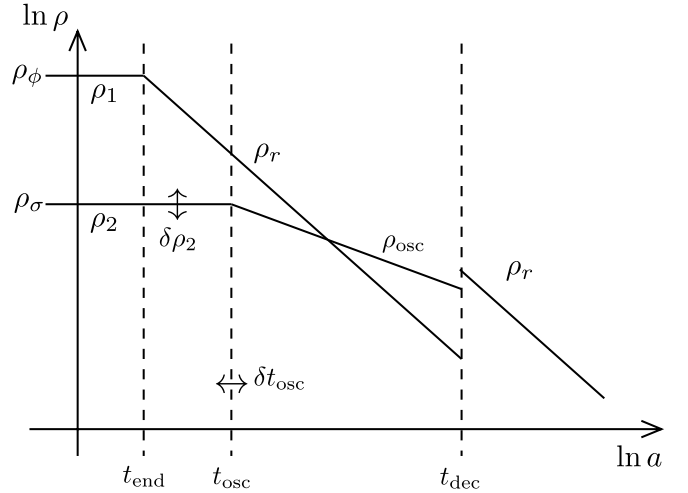


FIG. 1: Evolution of the energy density of the curvaton and the background radiation. The graph shows the evolution of the “oscillating curvaton”, which dominates the energy density of the Universe during oscillation. The crossing ($\rho_{\text{osc}} = \rho_r$) does not happen if $r_{\text{dec}} < 1$. As is discussed in the text, r_{dec} is determined by the model parameters.

Let us first consider the basic curvaton scenario. The initial perturbation of the curvaton σ is given by

$$\left. \frac{\delta \rho_\sigma}{\rho_\sigma} \right|_{\text{osc}} \simeq 2 \frac{\delta \sigma}{\sigma}. \quad (1)$$

This corresponds to the perturbation $\delta\rho_2$ in Fig.1. If the oscillation starts at $m_\sigma^2 \simeq H^2$, t_{osc} (in the figure) is exactly determined by the curvaton mass m_σ . Then there

¹ See also Ref. [5] for another application of the inflating curvaton.

is no perturbation which corresponds to δt_{osc} . Obviously, both $\delta\rho_2$ and δt_{osc} can source the curvaton perturbation, but in the basic scenario the source perturbation is almost entirely coming from $\delta\rho_2$.

The total curvature perturbation during the curvaton oscillation is given by

$$\zeta = (1 - f)\zeta_r + f\zeta_\sigma, \quad (2)$$

where we introduced the ratio $f \equiv 3\rho_\sigma/(4\rho_r + 3\rho_\sigma)$. Here $\zeta_r \equiv \delta\rho_r/4\rho_r$ is negligible. To calculate the curvaton perturbation, what we need is the quantity ζ_σ at the beginning of the oscillation because ζ_σ , defined by

$$\zeta_\sigma \equiv -H \frac{\delta\rho_\sigma}{\dot{\rho}_\sigma} = \frac{1}{3} \frac{\delta\rho_\sigma}{\rho_\sigma}, \quad (3)$$

becomes constant during the (quasi)sinusoidal oscillation.

The new idea in this paper is that ζ_σ is dominated by the perturbation δt_{osc} . In the basic (non-hybrid) scenario, the perturbation caused by δt_{osc} does not dominate the total curvature perturbation.² The reason is as follows. The source of δt_{osc} in the basic curvaton scenario is δm_σ , and m_σ is not a constant when the potential is not quadratic. Assuming that the sinusoidal oscillation starts at $\eta = 1$, the perturbation at the beginning of the oscillation is given by

$$\delta t_{\text{osc}} \simeq -\frac{\delta\eta}{\dot{\eta}} \quad (4)$$

Here we consider

$$\delta\eta \simeq \frac{\partial\eta}{\partial\sigma}\delta\sigma \quad (5)$$

$$\dot{\eta} \simeq \frac{\partial\eta}{\partial\sigma}\dot{\sigma} + \frac{\partial\eta}{\partial H}\dot{H} \simeq -2\eta\frac{\dot{H}}{H^2}H = 4\eta H, \quad (6)$$

where, in contrast to the inflation phase, $\dot{H}/H^2 = -2$ in the radiation dominated Universe, which dominates the evolution of η . Assuming a polynomial function $\eta \propto \sigma^p$, we find $\partial\eta/\partial\sigma = p\eta/\sigma$ and

$$\delta t_{\text{osc}} \simeq \frac{p}{4H} \frac{\delta\sigma}{\sigma}, \quad (7)$$

where the quadratic potential gives $p = 0$, which always leads to $\delta t_{\text{osc}} = 0$. For $p \neq 0$, we find

$$\left. \frac{\delta\rho_\sigma}{\rho_\sigma} \right|_{\delta t_{\text{osc}}} \simeq \frac{\dot{\rho}_\sigma \times \delta t_{\text{osc}}}{\rho_\sigma} = -3H\delta t_{\text{osc}} \simeq \frac{3p}{4} \frac{\delta\sigma}{\sigma}. \quad (8)$$

The above perturbation is comparable to the perturbation sourced by $\delta\rho_2$ in the basic curvaton scenario. Therefore, this perturbation does not dominate the curvature perturbation, but may change its non-Gaussianity.

As the result, the deviation from the quadratic potential ($p \neq 0$) may cause $\delta t_{\text{osc}} \neq 0$, but this perturbation cannot dominate over the total curvature perturbation. In the original curvaton scenario, the above dynamics has been included in the function $g(\sigma_*)$ introduced in Ref. [11]. (See also Ref.[12, 13])

II. NON-INFLATING HYBRID CURVATON

To find a significant enhancement of the curvaton perturbation, consider a hybrid potential that is given by

$$V = \frac{\lambda}{4} (M^2 - \sigma^2)^2 + \frac{g^2}{2} \varphi^2 \sigma^2 + \frac{1}{2} m_\varphi^2 \varphi^2. \quad (9)$$

The potential during slow-roll is $V_0 \simeq \frac{\lambda}{4} M^4$.

The hybrid potential usually leads to generation of cosmic strings at the phase transition ($\sigma = 0 \rightarrow \sigma \neq 0$), but in this curvaton scenario the strings do not lead to a serious cosmological problem because the typical energy scale is much lower than the cosmological bound.

Suppose that the “rolling field” (φ) rolls from far away ($\varphi_{\text{ini}} \gg \varphi_c$) and the sinusoidal oscillation of the “waterfall field” (σ) starts suddenly after the phase transition. In this model, t_{osc} is determined by the “rolling field” reaching the waterfall at $\varphi(t_{\text{osc}}) = \varphi_c$, where $\varphi_c \equiv (\sqrt{\lambda}/g)M$ is the critical value which triggers the phase transition. Therefore, the perturbation δt_{osc} [14] is given by

$$\delta t_{\text{osc}} \simeq \frac{\delta\varphi}{\dot{\varphi}}, \quad (10)$$

where $\dot{\varphi}$ denotes the velocity in the radiation-dominated Universe evaluated at the waterfall. On the other hand, the initial value of the curvaton oscillation is given by $\rho_\sigma(t_{\text{osc}}) = V_0 = \frac{\lambda}{4} M^4$, which means that the conventional source of the perturbation ($\delta\rho_2$ in Fig. 1) vanishes in this model. For simplicity, assume that the energy density of the curvaton oscillation evolves as $\rho_\sigma \propto a^{-3}$ just after the phase transition.

In this scenario, the perturbation of the “rolling field” ($\delta\varphi \neq 0$) is converted into δt_{osc} to cause the perturbation $\delta\rho_\sigma$. In this model, the transfer of the perturbation $\delta\varphi \rightarrow \delta t_{\text{osc}} \rightarrow \delta\rho_\sigma$ sources the curvaton perturbation $\zeta_\sigma \equiv \delta\rho_\sigma/3\rho_\sigma$.

The modulation of the phase transition ($\delta t_{\text{osc}} \simeq \delta\varphi/\dot{\varphi}$) causes the perturbation of the curvaton density;

$$\frac{\delta\rho_\sigma}{\rho_\sigma} \simeq \frac{\dot{\rho}_\sigma \delta t_{\text{osc}}}{\rho_\sigma} \simeq -3H \frac{\delta\varphi}{\dot{\varphi}}, \quad (11)$$

where $\dot{\rho}_\sigma = -3H\rho_\sigma$ is assumed for the oscillation. If φ rolls slowly before the phase transition, there is a significant enhancement of the curvaton perturbation. Note that, in the above equation, $\dot{\varphi}$ is not the conventional slow-roll velocity which is usually given by

² Modulated interaction may cause the curvature perturbation to be dominated by δt_{osc} [9, 10]. Such modulated (inhomogeneous) interaction is not considered in this paper.

$\dot{\varphi}_s \equiv -V_\varphi/3H$. This field has a significant acceleration $\ddot{\varphi} \neq 0$ due to $\dot{H}/H^2 = -2$ in the radiation-dominated Universe. Moreover, the typical expansion given by $\delta\varphi = \dot{\varphi}\delta t + \frac{1}{2}\ddot{\varphi}(\delta t)^2 + \dots$ shows that the relation between δt and $\delta\varphi$ is obviously non-linear. This kind of non-linearity does not appear in the case of the standard quasi-de Sitter inflation (with φ the inflaton), since $\ddot{\varphi} \simeq 0$ is usually assumed for φ to undergo slow-roll. However, $\ddot{\varphi} \simeq 0$ is not true in the radiation-dominated background.

For later convenience, we consider the approximation;

$$\dot{\varphi}(t_{\text{osc}}) \simeq \alpha \dot{\varphi}_s \equiv -\alpha \frac{V_\varphi}{3H_{\text{osc}}}, \quad (12)$$

where $H_{\text{osc}}^2 \equiv \rho_r/3M_p^2 > V_0/3M_p^2$ denotes the Hubble parameter at the phase transition. Here M_p is the reduced Planck mass. The deviation from slow-roll is measured by $\alpha \neq 1$. We assume $\alpha \sim \mathcal{O}(1)$ to avoid extreme situations.³ In some extreme models the non-Gaussianity could be affected, but in the present scenario the effect is at most $\mathcal{O}(1)$. In the radiation-dominated Universe and for the quadratic potential, $\alpha = 3/5$ has been derived in Ref. [13]. The opposite ($\rho_r < V_0$) corresponds to the inflating curvaton scenario. An intermediate scenario where $H_{\text{osc}}^2 \sim V_0/3M_p^2$ is also discussed in this paper.

Considering the oscillation of σ as in the basic curvaton scenario, the density perturbation of the curvaton is

$$\zeta_\sigma \equiv -H \frac{\delta\rho_\sigma}{\dot{\rho}_\sigma} = \frac{1}{3} \frac{\delta\rho_\sigma}{\rho_\sigma}. \quad (13)$$

From Eq.(11), we find

$$\zeta_\sigma \simeq - \left(H \frac{\delta\varphi}{\dot{\varphi}} \right)_{\text{osc}} \quad (14)$$

$$= \left(\frac{1}{\alpha} \frac{3H^2}{m_\varphi^2} \frac{\delta\varphi}{\varphi} \right)_{\text{osc}} \quad (15)$$

$$\equiv \left(\frac{1}{\alpha\eta} \frac{\delta\varphi}{\varphi} \right)_{\text{osc}}, \quad (16)$$

where $\eta \equiv m_\varphi^2/3H^2$ denotes the slow-roll parameter of the rolling field φ . For later convenience, we introduce a new parameter β that is defined by

$$\beta \equiv \frac{m_\varphi^2}{m_0^2}, \quad (17)$$

where $m_0^2 \equiv \frac{V_0}{M_p^2}$ is the possible soft-breaking mass that usually appears in supergravity when the F-term is given by $|F| \sim V_0^{1/2}$. It is evident that $\beta \sim 1$ is motivated by supergravity, and $\beta \not\sim 1$ measures the deviation from the naive setup.⁴ $\eta \ll 1$ and $\beta \sim 1$ are possible at the same time if the Universe is dominated by the radiation [17].

A. Slow-roll and slow-decay

Let us consider the spectrum $\mathcal{P}_{\delta\varphi}^{1/2} \simeq H_1/2\pi$ for the slow-rolling field, where H_1 is the Hubble parameter during the primordial inflation. Defining the energy ratio at the beginning of the σ oscillation as

$$r_{\text{osc}} \equiv \frac{V_0}{\rho_r(t_{\text{osc}})} = \frac{\lambda M^4}{12M_p^2 H_{\text{osc}}^2} \ll 1,$$

we find at t_{osc} ;

$$\eta = \beta r_{\text{osc}}. \quad (18)$$

Again, $\eta \ll \beta$ is possible when $r_{\text{osc}} \ll 1$. The spectrum of the curvaton perturbation is given by

$$\mathcal{P}_{\zeta_\sigma}^{1/2} \simeq \frac{1}{\alpha\beta r_{\text{osc}}} \left(\frac{H_1/2\pi}{\varphi} \right)_{\text{osc}} \simeq \frac{g}{2\pi\alpha\beta r_{\text{osc}}\sqrt{\lambda}} \frac{H_1}{M}. \quad (19)$$

Here the evolution of $\delta\varphi$ is neglected because of the slow-roll assumption. Defining the gravitational decay constant as $\Gamma_g \simeq (\sqrt{\lambda}M)^3/M_p^2$, and considering the actual decay rate $\Gamma_\sigma \equiv \xi\Gamma_g \geq \Gamma_g$, where $\xi (\gtrsim 1)$ measures the deviation from the gravitational decay, we find that the Hubble parameter at the decay is given by

$$H_{\text{dec}} \simeq \Gamma_\sigma \simeq \xi \frac{\lambda^{3/2} M^3}{M_p^2}, \quad (20)$$

which leads to the density ratio

$$r_{\text{dec}} \equiv \frac{\rho_\sigma}{\rho} \Big|_{\text{dec}} = r_{\text{osc}} \times \left(\frac{H_{\text{osc}}}{H_{\text{dec}}} \right)^{1/2} \quad (21)$$

where the evolution is possible until $r_{\text{osc}} = 1$. Here the value of $r_{\text{osc}} < 1$ (at the beginning of the oscillation) is determined by the initial condition. Although the situation depends on the choice of the model, $\xi \gg 1$ may cause significant interaction between φ and the radiation, which could alter the potential. In this sense, the potential used in the above argument is the effective potential approximated at the point very close to the waterfall and it includes all these corrections.

³ Although it depends on situation, the assumption $\alpha \sim \mathcal{O}(1)$ may hold even if the field φ started oscillating near the waterfall. Here the breakdown of slow-roll occurs first, then fast-roll may follow. There could be a breakdown of fast-roll if $\beta > 1$ (β is defined in Eq. (17)), where φ starts oscillating but we assume the oscillation soon stops at the waterfall. Assuming that fast-roll breaks down at $\varphi = \varphi_b$, φ gains the kinetic energy $K_\Delta < \frac{1}{2}m_\varphi^2\varphi_b^2$. Comparing K_Δ with the (possible) slow-roll kinetic energy at the waterfall; $K_s \simeq \left[\frac{m_\varphi^2\varphi_c}{3H} \right]^2$, we find approximately $\alpha < \varphi_b/\varphi_c$. Note that in Eq. (11) the conventional slow-roll is not assumed.

⁴ D-term inflation may avoid $\beta \sim 1$ [15, 16]. $\beta > 1$ is inevitable if the usual supersymmetry breaking sector (which is not related to the curvaton potential) gives the mass term with $\beta > 1$. In both cases β is highly model-dependent.

The spectral index is $n - 1 \simeq -2\epsilon_1 + 2\eta$ and the non-Gaussianity is $f_{NL} = \frac{4}{5}r_{\text{dec}}^{-1}$ for $r_{\text{dec}} \ll 1$, as in the usual curvaton scenario. Here $\epsilon_1 \equiv \dot{H}/H^2$ and $\eta \equiv m_\varphi^2/3H^2$ are defined at the horizon exit.

B. Slow-roll and fast-decay

As a second example, we consider the fast decay scenario. Again, we find the spectrum of the perturbation given by

$$\mathcal{P}_{\zeta_\sigma}^{1/2} \simeq \frac{1}{\alpha\beta r_{\text{osc}}} \left(\frac{\mathcal{P}_{\delta\varphi}^{1/2}}{\varphi} \right)_{\text{osc}} \simeq \frac{g}{2\pi\alpha\beta r_{\text{osc}}\sqrt{\lambda}} \frac{H_1}{M}. \quad (22)$$

Here the fast-decay assumption leads to $H_{\text{dec}} \sim H_{\text{osc}}$. To quantify the ratio, we introduce a parameter defined by

$$P_d \equiv \frac{H_{\text{dec}}}{H_{\text{osc}}} \leq 1, \quad (23)$$

where $P_d = 1$ corresponds to the instant decay. We find

$$r_{\text{dec}} \equiv \frac{\rho_\sigma}{\rho} \Big|_{\text{dec}} = \frac{\rho_\sigma}{\rho_r} \Big|_{\text{osc}} \times \left(\frac{H_{\text{osc}}}{H_{\text{dec}}} \right)^{1/2} = \frac{r_{\text{osc}}}{\sqrt{P_d}}. \quad (24)$$

As in the usual curvaton scenario, the phase transition at $r_{\text{dec}} \ll 1$ leads to enhanced non-Gaussianity. The model predicts that, whenever $r_{\text{osc}} \lesssim 1$ is natural in the hybrid curvaton scenario, the instant decay may affect non-Gaussianity. This is not mandatory in the slow-roll scenario, but could be mandatory if slow-roll breaks down before the waterfall. We are going to consider this possibility in the next section, although a more exact calculation requires numerical study.

C. Breakdown of slow-roll and fast decay

Since the radiation energy density is decreasing while m_φ is constant, the natural idea is that $\eta \sim 1$ triggers the significant variation of φ and eventually it causes the phase transition. Because the contribution of φ to the scalar potential is small (i.e., $m_\varphi^2 \varphi^2 \ll V_0$), the kinetic energy of φ is not significant even if the slow-roll is violated before the waterfall.⁵ To avoid an extreme situation, our modest assumption in this section is $\alpha \sim 1$. On the other hand, the perturbation $\delta\varphi$ leaves horizon during the primordial inflation, when the slow-roll parameters of φ are supposed to be much smaller than unity. We thus find that the spectral index is small, $n - 1 \ll 1$.

The problem in the slow-roll scenario is that, since the variation of φ (i.e., $\Delta\varphi \equiv \varphi(t_{\text{ini}}) - \varphi_c$) is negligible, we

always have to assume careful tuning of the initial condition $\varphi(t_{\text{ini}}) \simeq \varphi_c$. This tuning is relaxed if slow-roll breaks down just before the phase transition. Moreover, since $\beta \sim 1$ is justified in supergravity, there is the *important prediction* that $r_{\text{osc}} \lesssim 1$ in the hybrid curvaton model.

To look into more details, consider the evolution in the radiation-dominated Universe. This immediately leads to the equation of motion

$$\ddot{\varphi} + 3H(t)\dot{\varphi} + (cH(t)^2 + m_\varphi^2)\varphi = 0, \quad (25)$$

where the Hubble parameter evolves as

$$H(t) = \frac{1}{2t}. \quad (26)$$

Before fast-roll, we assume $m_\varphi^2 \ll cH(t)^2$. Substituting $dt = 4td\tau/3$, we find for the radiation dominated Universe [12]

$$\frac{d^2\varphi}{d\tau^2} + \frac{2}{3}\frac{d\varphi}{d\tau} + \frac{3c}{9}\varphi \simeq 0, \quad (27)$$

which has an oscillating solution when

$$c \geq \frac{1}{4}. \quad (28)$$

According to Ref. [17], there can be a cancellation that leads to $c \ll 1$ during radiation domination. Therefore, a conceivable set-up of the model is $c \ll 1/4$ with fast-roll starting when $m_\varphi^2 = H(t_{\text{fast}})^2/4$. At this moment ($t = t_{\text{fast}}$), we find

$$\rho_r = 12m_\varphi^2 M_p^2 = 12\beta V_0. \quad (29)$$

To capture the evolution after the breaking of slow-roll, we introduce $\gamma \equiv t_{\text{osc}}/t_{\text{fast}} \geq 1$, which is determined by the evolution thereafter. $\gamma = 1$ means that the σ -oscillation starts immediately after the breaking of slow-roll. Note that a non-trivial evolution of the waterfall field can be described by $\gamma \gg 1$, which we will not consider. There could be a small-scale perturbation of $\delta\sigma$, but this is not the topic of this paper.

For $\gamma^2 < 12\beta$, the σ -oscillation starts in the radiation-dominated Universe. This means that r_{osc} is given by

$$r_{\text{osc}} = \frac{\gamma^2}{12\beta} < 1. \quad (30)$$

Assuming fast decay ($P_d \sim 1$) and $\beta \sim 1$, the slow-roll breakdown just before the waterfall may produce naturally the coincidence $r_{\text{dec}} \lesssim 1$. Therefore, the scenario gives rise to $r_{\text{dec}} \lesssim 1$, which can affect non-Gaussianity.

One may suspect that a large initial value ($\varphi \sim M_p$) can easily cause a long-time rolling that leads to the curvaton domination before the phase transition. This speculation is true. In this respect, the scenario more or less depends on the initial condition. $\varphi(t_{\text{ini}}) \gg \varphi_c$ with $\beta \sim 1$ may lead to the opposite scenario, in which domination

⁵ The opposite limit leads to oscillations of the same type as in locked inflation [7], which is not considered in this paper.

by the curvaton starts *before* the phase transition. This scenario of the inflating curvaton [4] will be discussed in the next section.

In the above scenario with $\beta \sim 1$, long-time oscillation of the waterfall field is not needed for the curvaton domination. The waterfall field σ can decay after a few oscillations, *in contrast to the standard curvaton scenario*, in which a small decay rate is needed for the curvaton domination.

The spectrum of the curvature perturbation is obtained in the same way as in the previous (slow-roll) scenario;

$$\mathcal{P}_{\zeta}^{1/2} \simeq \frac{1}{\alpha\beta r_{\text{osc}}} \left(\frac{\mathcal{P}_{\delta\sigma}^{1/2}}{\varphi} \right)_{\text{osc}} \simeq \frac{1}{\alpha\beta r_{\text{osc}}} \left(\frac{\mathcal{P}_{\delta\sigma}^{1/2}}{\varphi} \right)_{\text{ini}}, \quad (31)$$

where the ratio $\delta\varphi/\varphi$ behaves like a constant when $V(\varphi)$ is quadratic. Otherwise, the result is highly model-dependent. Note that the deviation from $\alpha \sim 1$ might be significant at the waterfall if the evolution is already far from slow-roll (i.e., when $\beta \gg 1$). In this case, the non-linear evolution of φ and $\delta\varphi$ is significant and it requires further study. Here, considering a model with the modest evolution $\alpha \sim 1$ and $\beta \sim 1$ (as motivated by supergravity), we expect $r_{\text{osc}} \sim 0.1$.

III. INFLATING HYBRID CURVATON

It is possible to realize the inflating curvaton scenario if the oscillation starts after curvaton domination. This scenario always gives $r_{\text{osc}} = 1$ by definition, and helps to explain the spectral index and its running by reducing the number of e-foldings required for the primordial inflation [4]. The number of e-foldings during the curvaton inflation (N_2) is determined by the initial condition $\varphi(t_{\text{ini}})$ and β ;

$$N_2 = \frac{1}{\beta} \ln \frac{\varphi(t_{\text{ini}})}{\varphi_c}, \quad (32)$$

where t_{ini} denotes the beginning of the curvaton inflation. In this section we assume $\mathcal{O}(1)$ couplings for the hybrid potential ($\lambda \simeq g \simeq 1$) just for simplicity. Considering the bound $\varphi(t_{\text{ini}}) < M_p$ and $M > 1\text{TeV}$ (i.e., $\varphi_c > 1\text{TeV}$), we find from Eq. (32) that the maximum number of N_2 is given by

$$N_2 < 35\beta^{-1}. \quad (33)$$

Slow-roll ($\beta \ll 1$) requires $\varphi(t_{\text{ini}}) \sim \varphi_c$ if we demand $N_2 < 60$. We are not excluding this possibility, but it would be interesting to relax this tuning using a fast-roll potential ($\eta \sim \beta \sim 1$). In this case we find

$$\delta N_2 = \frac{1}{\beta} \frac{\delta\varphi}{\varphi} \sim \frac{\delta\varphi}{\varphi}. \quad (34)$$

Since the ratio $\delta\varphi/\varphi$ does not evolve in the case of a quadratic potential, we obtain

$$\mathcal{P}_{\zeta}^{1/2} \sim \frac{H_1}{2\pi\varphi_c e^{\beta N_2}}, \quad (35)$$

where the relation $\varphi(t_{\text{ini}}) \simeq \varphi_c e^{\beta N_2}$ was used. Here $\mathcal{P}_{\zeta}^{1/2}$ denotes the observable spectrum from the CMB.

If we consider the stochastic initial condition $H_1 > M$ [4], we find $\mathcal{P}_{\zeta}^{1/2} \gg e^{-\beta N_2}$. Here $\varphi_c \simeq M$ is used because $\mathcal{O}(1)$ couplings are assumed. Therefore, the stochastic condition leads to

$$N_2 > 13\beta^{-1} \sim 13. \quad (36)$$

This result is in sharp contrast to the PNGB inflating curvaton [4], in which the stochastic condition puts an *upper* bound for N_2 .

An interesting application of this scenario is that M can be related to the dynamical scale of supersymmetry breaking while $\beta \sim 1$ is due to the supergravity action. Considering $M \sim \Lambda_s \sim 10^{11}\text{GeV}$, we find (The upper bound is different from Eq.(33) because the scale of M is different.)

$$13 < N_2 < 16, \quad (10^{16}\text{GeV} < \varphi(t_{\text{ini}}) < M_p) \quad (37)$$

and for $M \sim 10^6\text{GeV}$ we find

$$13 < N_2 < 27, \quad (10^{11}\text{GeV} < \varphi(t_{\text{ini}}) < M_p). \quad (38)$$

As in the standard curvaton scenario, the spectral index is given by

$$n - 1 = -2\epsilon_1 + 2\eta, \quad (39)$$

where ϵ_1 (for the primordial inflaton) and η (for the slow-roll curvaton field) are calculated at the horizon exit. If the primordial inflation is of the chaotic type with potential $V(\phi) \simeq A\phi^p$, then $\epsilon_1 \simeq p/4N_1$. According to observations, $n - 1 = -0.037 \pm 0.014$ [18], which leads to $N_1 \simeq 14p$ for negligible η . In the oscillating curvaton scenario ($N_2 = 0$) the spectral index excludes $p < 4$, and gives for the running of the spectral index $n' \equiv (n - 1)/N_1 \simeq 0.0007$. The situation is quite different in the inflating curvaton scenario. Because of the second inflation $N_2 \neq 0$, $p < 4$ is not ruled out, while the running of the spectral index is enhanced when $N_1 < 60$, which can have observational impact [4].

IV. CONCLUSIONS

One of the advantages of the curvaton idea is that the sector, which gives rise to primordial inflation (inflaton sector), can be decoupled from the sector which gives rise to the curvature perturbation (curvaton sector). This can naturally accommodate the stability of the hierarchy in scales, which is a feature of the curvaton scenario. Moreover, in the curvaton scenario, the inflaton sector

does not even need to feature a fundamental scalar field at all.⁶ In that sense, the curvaton scenario liberates the inflaton sector from many serious constraints [25].

Although the inflaton sector has been investigated in the context of various theoretical models, including the hybrid-type potential for the inflaton field, the curvaton sector is usually considered merely with the simple quadratic potential. This gives somewhat “biased” information about the possibilities for inflationary cosmology. In that sense, our paper is an effort to expand the state of the art. Furthermore, the hybrid potential, which is the target in this paper, is well motivated in particle physics without being necessarily tied to the inflaton. For example, the hybrid structure may appear in GUTs or brane-brane interactions [24, 26] while it can well explain the differences in scales [26]. Finally, in the hybrid curvaton, the source mechanism of the curvature perturbation is utterly different from the original curvaton scenario. In that sense the basic formulation is brand-new.

To be more specific, we considered in this paper a new possibility of the curvaton scenario; we studied oscillating, inflating and non-slow-roll scenarios for the hybrid curvaton. For the slow-roll scenarios we found significant enhancement of the curvature perturbation, while in the non-slow-roll scenarios we found that the tuning requirements for the initial condition are relaxed. The inflating curvaton predicts distinguishable running of the spectral index, which cannot be attained in the usual inflation scenario except for a few extended models.⁷ There is a variety of models possible for the hybrid curvaton. For example, one can consider the curvaton analogues of smooth hybrid inflation [21] or shifted hybrid inflation [22].

Since the purpose of this paper is not the model building but the basic formulation of the brand-new idea,

⁶ The use of $f(R)$ gravity in the inflationary cosmology was pioneered by Starobinsky [23].

⁷ Some extended models can attain significant running even for single-field inflaton models. for instance, the running mass

we avoided highly model-dependent arguments and focused on the simplest model that is useful to explain the essential idea of the scenario. We have considered $\alpha = 3(H\dot{\varphi}/V_{\varphi})_{\text{osc}} \sim 1$ because it is the natural choice. However, the scenario with $\alpha \gg 1$ is interesting as well. A large α would reduce the spectrum of the curvature perturbation, but the non-linear evolution might enhance non-Gaussianity, in a way that has not been considered yet. However, the calculation requires numerical study. We have also considered $\beta \sim 1$ (β is defined in Eq. (17)) because this is well motivated by supergravity. β is generically a model-dependent parameter and, were it not for the supergravity motivation, there would be no reason to believe that $\beta \sim 1$. Finally, we considered that the oscillation starts immediately after the end of the slow-roll evolution to avoid the complexity related to the dynamics of the waterfall. However, in some cases this dynamics could be non-trivial, which may require numerical calculations. Indeed, the evolution of the waterfall is not easily understood by analytical calculation only [6] even without a radiation background.

V. ACKNOWLEDGMENT

We thank David H. Lyth for valuable discussions. KD is supported (in part) by the Lancaster-Manchester-Sheffield Consortium for Fundamental Physics under STFC grant ST/J000418/1. KK is partly supported by the Grant-in-Aid for the Ministry of Education, Culture, Sports, Science and Technology, Government of Japan Nos. 21111006, 22244030, 23540327, and by the Center for the Promotion of Integrated Science (CPIS) of Soken-dai (1HB5806020).

model [19] and the Type III Hilltop inflation model [20] predict such a large running of the spectral index.

-
- [1] D. H. Lyth and A. R. Liddle, *The primordial density perturbation*, Cambridge University Press, 2009.
 - [2] S. Mollerach, Phys. Rev. D **42** (1990) 313; A. D. Linde and V. F. Mukhanov, Phys. Rev. D **56**, 535 (1997) [astro-ph/9610219]; D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002) [hep-ph/0110002]; T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001) [Erratum-ibid. B **539**, 303 (2002)] [hep-ph/0110096]; K. Enqvist and M. S. Sloth, Nucl. Phys. B **626** (2002) 395 [arXiv:hep-ph/0109214].
 - [3] A. Mazumdar and J. Rocher, Phys. Rept. **497** (2011) 85 [arXiv:1001.0993 [hep-ph]].
 - [4] K. Dimopoulos, K. Kohri, D. H. Lyth and T. Matsuda, arXiv:1110.2951 [astro-ph.CO].
 - [5] K. Furuuchi and C. -M. Lin, arXiv:1111.6411 [hep-ph].
 - [6] D. H. Lyth, arXiv:1107.1681 [astro-ph.CO]; D. H. Lyth, JCAP **1107**, 035 (2011) [arXiv:1012.4617 [astro-ph.CO]].
 - [7] K. Dimopoulos and M. Axenides, JCAP **0506**, 008 (2005) [hep-ph/0310194].
 - [8] T. Matsuda, Phys. Rev. D **68**, 047702 (2003) [hep-ph/0302253].
 - [9] T. Matsuda, Class. Quant. Grav. **26**, 145011 (2009). [arXiv:0902.4283 [hep-ph]]; T. Matsuda, JCAP **0812**, 001 (2008). [arXiv:0811.1318 [hep-ph]].
 - [10] T. Matsuda, JHEP **0707**, 035 (2007). [arXiv:0707.0543 [hep-th]].
 - [11] D. H. Lyth, Phys. Lett. B **579** (2004) 239 [hep-th/0308110].
 - [12] D. H. Lyth, C. Ungarelli and D. Wands, Phys. Rev. D **67**, 023503 (2003) [astro-ph/0208055]; K. Dimopoulos, G. Lazarides, D. Lyth and R. Ruiz de Austri, Phys. Rev. D **68**, 123515 (2003) [hep-ph/0308015].
 - [13] M. Kawasaki, T. Kobayashi, F. Takahashi, [arXiv:1107.6011 [astro-ph.CO]].
 - [14] K. A. Malik, D. Wands, Class. Quant. Grav. **21**, L65-L72 (2004). [astro-ph/0307055].

- [15] P. Binetruy, G. R. Dvali, Phys. Lett. **B388**, 241-246 (1996). [hep-ph/9606342]; E. Halyo, Phys. Lett. **B387**, 43-47 (1996). [hep-ph/9606423].
- [16] T. Matsuda, JCAP **0311**, 003 (2003). [hep-ph/0302078]; T. Matsuda, Phys. Lett. **B423**, 35-39 (1998). [hep-ph/9705448].
- [17] D. H. Lyth and T. Moroi, JHEP **0405**, 004 (2004) [hep-ph/0402174].
- [18] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192**, 18 (2011).
- [19] E. D. Stewart, Phys. Lett. B **391**, 34 (1997) [hep-ph/9606241]; E. D. Stewart, Phys. Rev. D **56**, 2019 (1997) [hep-ph/9703232]; S. M. Leach, I. J. Grivell and A. R. Liddle, Phys. Rev. D **62**, 043516 (2000) [astro-ph/0004296].
- [20] K. Kohri, C. -M. Lin and D. H. Lyth, JCAP **0712**, 004 (2007) [arXiv:0707.3826 [hep-ph]].
- [21] G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D **52** (1995) R559 [arXiv:hep-ph/9506325].
- [22] R. Jeannerot, S. Khalil, G. Lazarides and Q. Shafi, JHEP **0010** (2000) 012 [arXiv:hep-ph/0002151]; R. Jeannerot, S. Khalil and G. Lazarides, JHEP **0207** (2002) 069 [arXiv:hep-ph/0207244].
- [23] A. A. Starobinsky, Phys. Lett. B **91** (1980) 99.
- [24] C. Herdeiro, S. Hirano and R. Kallosh, JHEP **0112**, 027 (2001) [hep-th/0110271].
- [25] K. Dimopoulos and D. H. Lyth, Phys. Rev. D **69** (2004) 123509 [arXiv:hep-ph/0209180].
- [26] J. Garcia-Bellido, hep-ph/0002256.